SOME PROPERTIES OF THE HEAT-TRANSFER PROCESS IN A COMPRESSED GAS WITH CONSIDERATION OF THERMAL FLUX RELAXATION

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Some solutions of the gas dynamics equations with thermal conductivity are examined with consideration of thermal flux relaxation and nonlinear dependence of the thermal conductivity coefficient and thermal flux relaxation time on temperature and density of the medium.

1. In describing intense heat exchange, in place of the Fourier law, which postulates proportionality of the thermal flux to the temperature gradient, a mathematical model can be used which considers thermal flux relaxation and relies on the equation

$$\tau \frac{\partial \mathbf{W}}{\partial t} + \mathbf{W} = -\varkappa \operatorname{grad} T, \qquad (1)$$

which dates back to Maxwell [1]. For constant $\varkappa$ and τ it is widely used for description of heat transport in rarefied gases, in hereditarily elastic materials, and in thermoelasticity problems (see [2-9] and bibliography therein). Results obtained with this expression agrees well with experimental data on the distribution of thermal pulses in solids at low temperatures [10]. Recently, interest in Eq. (1) has increased because of its proposed use to describe electron thermal conductivity in high-temperature plasma [11-14]. It should be noted that in this case Eq. (1) becomes nonlinear, since $\varkappa$ and τ depend on the thermodynamic parameters of the plasma. For example, for a completely ionized plasma, over a quite wide parameter range one may take $\varkappa \sim T^{5/2}$, $\tau \sim T^{3/2}/\rho$.

Many studies have investigated heat transport in a nonmoving medium in the linear case (at $\varkappa = \text{const}$, $\tau = \text{const}$) (see, for example, [2-9, 15, 16] and bibliography therein). The temperature dependence of $\varkappa$ and τ was considered in [11-14, 17].

2. In real physical problems the interaction of thermal and gas dynamic problems very often plays an important role. In the case of conventional quasilinear thermal conductivity (Fourier law at $\varkappa = \varkappa(T, \rho)$) properties of gas dynamic flows with consideration of heat transfer have been studied quite thoroughly (see [18-24]). When Eq. (1) is used in place of the Fourier law, the question naturally arises of the degree to which results obtained previously are transferrable to such a description of heat transfer.

The present study will consider some solutions of the system of gas dynamics equations with thermal conductivity in the form of Eq. (1), which can be written in mass Lagrangian variables in the one-dimensional planar case in the following form:

$$\begin{gathered} \frac{\partial}{\partial t}\left(\frac{1}{\rho}\right) = \frac{\partial v}{\partial m};\ \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial m}; \\ \frac{\partial \varepsilon}{\partial t} = -p\frac{\partial v}{\partial m} - \frac{\partial W}{\partial m};\ \tau\frac{\partial W}{\partial t} + W = -\tilde{\varkappa}\frac{\partial T}{\partial m}. \end{gathered} \qquad (2)$$

We will consider the gas ideal, $p = \rho RT$, $\varepsilon = c_V T = RT/(\gamma - 1)$; as assume that the thermal conductivity and thermal flux relaxation time depend on temperature and density in the following manner: $\tilde{\varkappa} = \varkappa_0 T^{a}\rho^{b+1}$, $\tau = \tau_0 T^{a_1}\rho^{b_1}$; $\varkappa_0$, τ_0, a, b, a_1, b_1 are constants; for a completely ionized plama $a = 5/2$, $a_1 = 3/2$, $b = 0$, $b_1 = -1$.

3. We will consider solutions of system (2) in the form of traveling waves, propagating with a constant mass velocity D. Just as in [21, 25], we limit ourselves to the case of a constant background with zero temperature, i.e., we assume a background with $T = 0$, $v = v_0$,

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 2, pp. 232-240, August, 1986. Original article submitted May 17, 1985.

0022-0841/86/5102-0921$12.50

$\rho = \rho_0$, $p = 0$, $W = 0$. Assuming that the solution depends only on the combination $Dt - m$, we transform to the dimensionless variable ξ and new dimensionless functions:

$$\xi = (Dt - m)\varkappa_0^{-1} D^{1-2a} R^{a+1} \rho_0^{2a-b-1};\ \bar{\eta}(\xi) = \rho_0/\rho(m,\ t);$$
$$\bar{v}(\xi) = v(m,\ t)\rho_0 D^{-1};\ \bar{p}(\xi) = p(m,\ t)\rho_0 D^{-2};\tag{3}$$
$$\bar{W}(\xi) = W(m,\ t)\rho_0^2 D^{-3};\ \bar{T}(\xi) = T(m,\ t)\rho_0^2 R D^{-2}.$$

From Eq. (2) we obtain a system of ordinary differential equations:

$$\frac{d\bar{\eta}}{d\xi} = -\frac{d\bar{v}}{d\xi};\ \frac{d\bar{v}}{d\xi} = \frac{d\bar{p}}{d\xi};\ \frac{1}{\gamma - 1}\frac{d\bar{T}}{d\xi} = \bar{p}\frac{d\bar{v}}{d\xi} + \frac{d\bar{W}}{d\xi};$$
$$\beta_0 \bar{T}^{a_1}\bar{\eta}^{-b_1}\frac{d\bar{W}}{d\xi} + \bar{W} = \bar{T}^{a}\bar{\eta}^{-b-1}\frac{d\bar{T}}{d\xi},\tag{4}$$

where β_0 is a dimensionless parameter:

$$\beta_0 = \tau_0\varkappa_0^{-1} D^{2a_1-2a+2} R^{a-a_1+1} \rho_0^{2a-b-1+2a_1+b_1}.$$

With consideration of the background values system (4) can be reduced to three algebraic expressions

$$\bar{v} = \bar{v}_0 + 1 - \bar{\eta};\ \bar{T} = \bar{\eta}(1 - \bar{\eta});\ \bar{W} = -\frac{\gamma - 1}{2(\gamma + 1)}(1 - \bar{\eta})\left(\bar{\eta} - \frac{\gamma - 1}{\gamma + 1}\right)\tag{5}$$

and a single ordinary differential equation

$$\frac{d\bar{\eta}}{d\xi} = \frac{\frac{\gamma + 1}{2(\gamma - 1)}(1 - \bar{\eta})\left(\bar{\eta} - \frac{\gamma - 1}{\gamma + 1}\right)}{\beta_0\frac{\gamma + 1}{\gamma - 1}\bar{\eta}^{a_1 - b_1}(1 - \bar{\eta})^{a_1}\left(\bar{\eta} - \frac{\gamma}{\gamma + 1}\right) + \bar{\eta}^{a-b-1}(1 - \bar{\eta})^{a}(1 - 2\bar{\eta})}.\tag{6}$$

We will consider in greater detail the case corresponding to a totally ionized plasma. At $a = 5/2$, $a_1 = 3/2$, $b = 0$, $b_1 = -1$ the parameter $\beta_0 = \tau_0\varkappa_0^{-1}R^2$, and Eq. (6) reduces to the form

$$\frac{d\bar{\eta}}{d\xi} = \frac{\gamma + 1}{2[(\gamma + 1)\beta_0 + 2(\gamma - 1)]}\,\frac{\bar{\eta} - \frac{\gamma - 1}{\gamma + 1}}{(1 - \bar{\eta})^{1/2}\bar{\eta}^{3/2}(\bar{\eta} - \bar{\eta}_1)(\bar{\eta} - \bar{\eta}_2)},\tag{7}$$

where

$$\bar{\eta}_{1,2} = \frac{[\gamma\beta_0 + 3(\gamma - 1)] \pm \sqrt{\gamma^2\beta_0^2 + 2\beta_0(\gamma - 1)(\gamma - 2) + (\gamma - 1)^2}}{2[(\gamma + 1)\beta_0 + 2(\gamma - 1)]}.\tag{8}$$

We note that the radicand in Eq. (8) is always positive for $\gamma > 1$ and $0 \leqq \bar{\eta}_{1,2} \leqq 1$. Analysis of various possibilities for expanding the roots $\eta_{1,2}$ in $(\gamma - 1)/(\gamma + 1)$ shows that for $1 < \gamma < 3$, $0 < \beta_0 < 2(3 - \gamma)/(\gamma + 1)$ the form of the integral curves of Eq. (7) is that shown in Fig. 1a. Figure 1b shows integral curves for $\beta_0 = 0$ and γ within the same range. If we assume that η_1 corresponds to the sign (+) in Eq. (8) while η_2 corresponds to (−), then as $\beta_0 \to 0$ the quantity η_1 tends to unity, while $\eta_2 \to 1/2$.

The form of the integral curves for the case $1 < \gamma < 3$, $\beta_0 = 2(3 - \gamma)/(\gamma + 1)$ is shown in Fig. 2a. Figure 2b shows the corresponding curve for $\gamma = 3$, $\beta_0 = 0$. Integral curves for the case $\beta_0 > (3 - \gamma)/(\gamma + 1)$, $\beta_0 > 0$, in particular, for any values $\beta_0 > 0$ at $\gamma \geqq 3$ are shown in Fig. 3a. As in the previous figures, Fig. 3b shows curves for $\beta_0 = 0$, $\gamma > 3$.

4. In the case of the diffusion approximation (Fourier law) the integral curves undergo a rotation at the point where the velocity of the traveling wave D becomes equal to the isothermal mass Lagrangian speed of sound $c = \rho\sqrt{RT}$ (see Fig. 1b, 2b, 3b). At $\gamma \geqq 3$ the integral curves reach values behind the shock wave $\eta = (\gamma - 1)/(\gamma + 1)$ before the rotation point and there exist continuous solutions, which as $\xi \to -\infty$ take on the value $\eta = 1$, while as $\xi \to +\infty$, $\eta = (\gamma - 1)/(\gamma + 1)$ (in the future we will term such solutions "solutions with shock wave type structure").

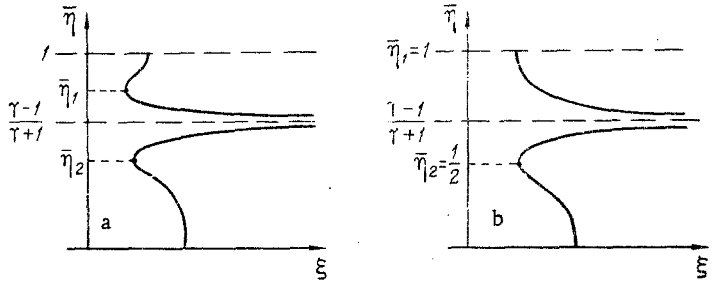

Fig. 1. Qualitative character of integral curves of Eq. (7) for cases: a) $1 < \gamma < 3$, $0 < \beta_0 < 2(3 - \gamma)/(\gamma + 1)$; b) $1 < \gamma < 3$, $\beta_0 = 0$.

The more complex behavior of the solutions of Eq. (7) as compared to the case $\beta_0 = 0$ is related to the fact that in a gas with heat transport in the form of Eq. (1) there exist two propagation speeds for small perturbations (see, for example, [7, 8, 25]). We will find the characteristics of Eq. (2). To do this, we rewrite the expression in the form

$$\frac{\partial \mathbf{u}}{\partial t} + A\frac{\partial \mathbf{u}}{\partial m} = \mathbf{B},$$

where

$$\mathbf{u} = \begin{Vmatrix} \rho \\ v \\ T \\ W \end{Vmatrix}; \quad A = \begin{Vmatrix} 0 & \rho^2 & 0 & 0 \\ TR & 0 & \rho R & 0 \\ 0 & \rho T(\gamma - 1) & 0 & (\gamma - 1)/R \\ 0 & 0 & \tilde{\varkappa}/\tau & 0 \end{Vmatrix};$$

$$\mathbf{B} = \begin{Vmatrix} 0 \\ 0 \\ 0 \\ -W/\tau \end{Vmatrix}.$$

Equating to zero the determinant $\det(A - \lambda E)$, where E is a unit matrix, we obtain

$$\lambda^4 - \lambda^2(\tilde{\varkappa}(\gamma - 1)/(R\tau) + \gamma\rho^2 RT) + (\gamma - 1)\rho^2\tilde{\varkappa}T/\tau = 0$$

or

$$\lambda^4 - \lambda^2(c_1^2 + \gamma c^2) + c^2 c_1^2 = 0, \qquad (9)$$

where $c_1 = \sqrt{\tilde{\varkappa}(\gamma - 1)/(R\tau)}$ is the mass velocity of heat propagation [4-6]. We write equations for the characteristics of system (2) in the form $dm/dt = \lambda_n$, where λ_n are roots of Eq. (9), having the sense of the propagation speed of small perturbations and weak discontinuities:

$$\lambda_1^2 = \frac{c_1^2 + \gamma c^2}{2} + \sqrt{\left(\frac{c_1^2 + \gamma c^2}{2}\right)^2 - c^2 c_1^2} =$$
$$= \frac{c_1^2 + \gamma c^2}{2} + \frac{1}{2}\sqrt{(c_1^2 - \gamma c^2)^2 + 4(\gamma - 1)c^2 c_1^2}, \qquad (10)$$
$$\lambda_2^2 = \frac{c_1^2 + \gamma c^2}{2} - \frac{1}{2}\sqrt{(c_1^2 - \gamma c^2)^2 + 4(\gamma - 1)c^2 c_1^2}.$$

It can be shown that the denominator on the right side of Eq. (6) vanishes if the speed of motion of the traveling wave D is equal to one of the velocities of small perturbation progation given by Eq. (10). To do this it is sufficient to reduce this denominator to the form

$$D^{-4}\frac{\beta_0}{\gamma - 1}\bar{\eta}^{a_1 - b_1}(1 - \bar{\eta})^{a_1}[D^4 - D^2(\gamma c^2 + c_1^2) + c^2 c_1^2].$$

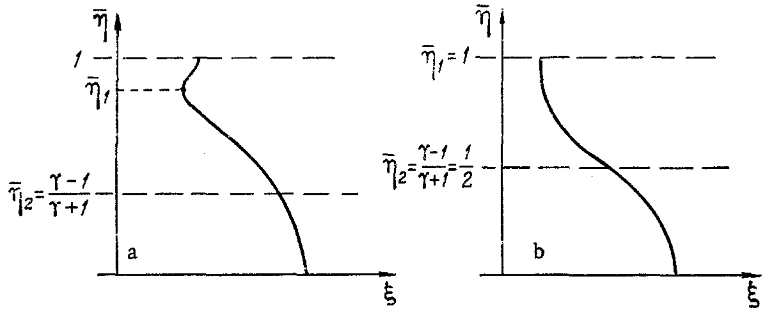

Fig. 2. Qualitative character of integral curves of Eq. (7) for cases: a) $1 < \gamma < 3$, $\beta_0 = 2(3 - \gamma)/(\gamma + 1)$; b) $\gamma = 3$, $\beta_0 = 0$.

As $\tau \to 0$ ($c_1 \to \infty$) the quantity $\lambda_1 \to \infty$, and by expanding the expressions for λ_2^2 in a series in powers of c^2/c_1^2 and neglacting terms of order $O(c^4/c_1^4)$, it is simple to obtain from Eq. (10) the fact that λ_2 tends to the isothermal speed of sound, $\lambda_2 \to c$.

5. The series of figures 1b, 2b, 3b have been taken from [21], which described in detail the properties of such traveling wave type solutions of the gas dynamics equations with ordinary nonlinear thermal conductivity (Fourier law). Therefore, we will consider in greater detail the differences of the solutions for the cases $\beta_0 = 0$ and $\beta_0 > 0$.

Considering that ξ increases with increasing t, it can easily be seen that departure from the constant initial background $\eta = 1$ along the integral curve is impossible for $\beta_0 > 0$. If a point belongs to the unperturbed background, then the gas parameters at that point can change only discontinuously. In examining the shock wave structure in the gas, where heat transfer in the form of Eq. (1) plays a role in the dissipation process, at these parameter values on the wave front there is a discontinuity in the physical quantities which would not occur in a gas with conventional nonlinear thermal conductivity.

For $\gamma > 3$ in a gas with conventional thermal conductivity the shock wave structure is continuous, the front being "spread" over an infinite width. In a gas with thermal conductivity in the form of Eq. (1) the shock wave structure also contains an infinitely long "tail," but includes a discontinuity in quantities on the wave front.

For $\gamma = 3$ the shock wave structure in a gas with ordinary thermal conductivity is continuous, and the front width is finite. In a gas with thermal flux relaxation the value $\eta = (\gamma - 1)/(\gamma + 1)$, corresponding to the state behind the shock wave is reached at a finite point only for $\beta_0 = 2(3 - \gamma)/(\gamma + 1)$, $1 < \gamma < 3$, while for larger β_0 the width of the front "spreading" is infinite. But nevertheless a discontinuity in the physical quantities still remains on the wave front.

At lower values of β_0 $\bar{\eta} \to (\gamma - 1)/(\gamma + 1)$ only as $\xi \to -\infty$, therefore in the final state $\bar{\eta} = (\gamma - 1)/(\gamma + 1)$ can be obtained only discontinuously. Considering the existence of an additional discontinuity from the initial state we find that the shock wave structure in this class includes two discontinuities in the physical quantities, related to the presence in the medium of two speeds of "sound," Eq. (10).

6. Solutions with two discontinuities were also obtained in numerical calculations of self-similar solutions of system (2) of the power type.

If initial conditions for system (2) are specified in the form

$$T(m,\ 0) = W(m,\ 0) = v(m,\ 0) = 0,\ \rho(m,\ 0) = \rho_0,$$

with boundary conditions

$$W(0,\ t) = W_0 t^g,\ v(0,\ t) = v_0 t^{\tilde{n}_0},$$

then at $\tilde{n}_0 = g/3$, $n_0\alpha - n_0 - n - g = 0$, $n_0\alpha_1 = 1$, where $n_0 = 2\tilde{n}_0$, $n = g/3 + 1$, the solution of the problem is self-similar. With the replacement of variables

$$m = S t^n W_0^{1/3} \rho_0^{2/3};\ T(m,\ t) = f(S)\, t^{n_0} R^{-1} W_0^{2/3} \rho_0^{-2/3};$$
$$W(m,\ t) = \omega(S)\, W_0 t^g;\ \rho(m,\ t) = \delta(S)\, \rho_0; \qquad (11)$$
$$v(m,\ t) = \alpha(S)\, t^{n_0/2} W_0^{1/3} \rho_0^{-1/3}$$

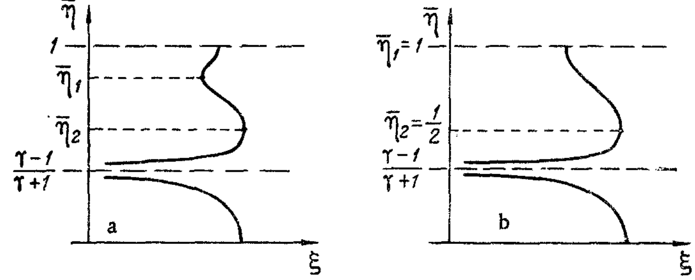

Fig. 3. Qualitative character of integral curves of Eq. (7) for cases: a) $\beta_0 > 0$, $\beta_0 > 2(3-\gamma)/(\gamma+1)$; b) $\gamma > 3$, $\beta_0 = 0$.

system (2) takes on the form

$$-nS\left(\frac{1}{\delta}\right)' = \alpha'; \quad \frac{1}{2}n_0\alpha - nS\alpha' = -(\delta' f + f'\delta);$$
$$\frac{n_0}{\gamma-1}f - \frac{n}{\gamma-1}Sf' - \delta f nS\left(\frac{1}{\delta}\right)' + \omega' = 0; \qquad (12)$$
$$\hat\tau(g\omega - nS\omega') + \hat\varkappa f' + \omega = 0,$$

where the prime denotes a derivative with respect to the new self-similar variable S; $\hat\varkappa = \hat\varkappa_0 f^{a}\delta^{b+1}$, $\hat\varkappa_0 = \varkappa_0 R^{-a-1}\rho_0^{-2a/3+b-1/3}W_0^{2(a-1)/3}$; $\hat\tau = \hat\tau_0 f^{a_1}\delta^{b_1}$, $\hat\tau_0 = \tau_0 R^{-a_1}\rho_0^{b_1-2a_1/3}W_0^{2a_1/3}$.

Initial and boundary conditions take on the form

$$\omega(0) = 1; \ \alpha(0) = \alpha_0 = v_0 W_0^{-1/3}\rho_0^{1/3};$$
$$f(\infty) = \omega(\infty) = \alpha(\infty) = 0; \ \delta(\infty) = 1.$$

If we now solve system (12) for the derivatives, then in the denominator of each expression we will have the main determinant of the system

$$\Delta = \hat\tau\frac{(nS)^2}{\gamma-1}[(nS)^2 - \gamma\delta^2 f] - \hat\varkappa[(nS)^2 - \delta^2 f].$$

We note that in the dimensionless self-evident variables of Eq. (11) the instantaneous velocity of motion of a profile point $\hat D = dm/dt = nS$, the spped of sound $\hat c = \delta\sqrt{f}$, and the speed of heat propagation $\hat c_1 = \sqrt{(\gamma-1)\hat\varkappa/\hat\tau}$, so that

$$\Delta = \frac{\hat\tau}{\gamma-1}[\hat D^4 - \hat D^2(\hat c_1^2 + \gamma\hat c^2) + \hat c^2\hat c_1^2].$$

It is evident that Δ vanishes if the velocity of motion of the wave is equal to the velocity of propagation of small perturbations.

For parameter values corresponding to a completely ionized plasma ($a = 5/2$, $a_1 = 3/2$, $b = 0$, $b_1 = -1$) the self-similarity conditions are satisfied at $g = 1$, $\tilde n_0 = 1/3$. Analysis of system (12) reveals that at $\tau_0 \neq 0$ there is a discontinuity in physical values on the wave front, just as in the travelling wave type solutions.

Steady state self-similar profiles of the physical quantities obtained by numerical calculations of system (2) with initial and boundary conditions at parameter values corresponding to a completely ionized plasma for $\alpha_0 = 1$, $\hat\varkappa_0 = 100$ are presented in Fig. 4. Two discontinuities in the solution are clearly visible in Fig. 4b. It is also evident that increase in the parameter $\hat\tau_0$ may produce a change in regime from TV-I to TV-II (see [18]), related to a decrease in the effective speed of thermal wave propagation (see [14]).

Thus, depending on the value of the thermal flux relaxation time τ qualitatively different regimes of heat propagation exist in a moving medium. An additional discontinuity in the physical quantities is introduced in the solution, related to the given method of describing heat transfer.

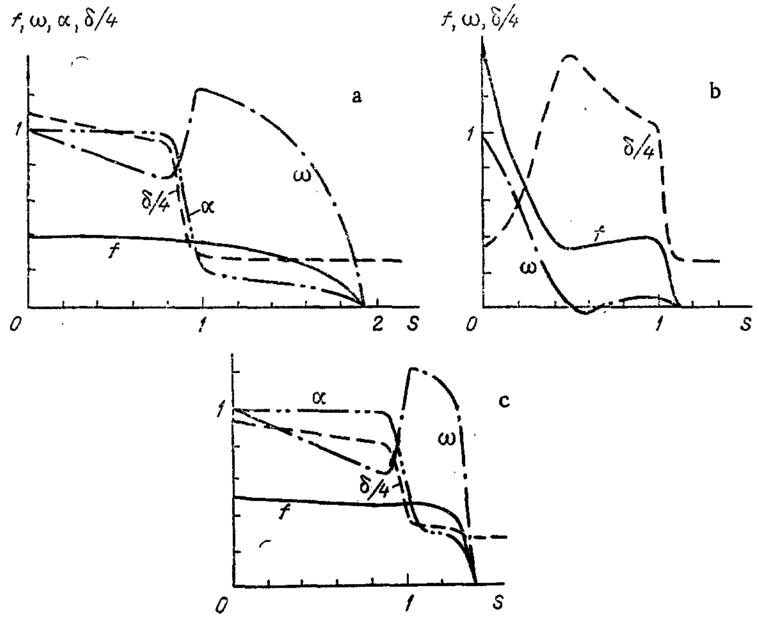

Fig. 4. Steady-state self-similar physical quantity profiles at $\alpha_0 = 1$, $\hat{\varkappa}_0 = 100$ and various $\hat{\tau}_0$: a) $\hat{\tau}_0 = 0$; b) 5; c) 1000.

NOTATION

x, spatial coordinate; m, mass Lagrangian coordinate; t, time; T, temperature; ρ, density; v, velocity; p, pressure; W, thermal flux; ϰ, thermal conductivity coefficient; $\tilde{\varkappa} = \rho\varkappa$, mass thermal conductivity coefficient; τ, thermal flux relaxation time; R, universal gas constant; γ, adiabatic index; $c_V = R/(\gamma - 1)$, specific heat at constant volume; ε, specific internal energy; c, isothermal speed of sound; c_1, speed of heat propagation; D, speed of traveling wave motion; S, self-similar independent variable; f, ω, δ, α, self-similar functions for temperature, thermal flux, density, and velocity.

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PERTURBATION-WAVE PROPAGATION IN PETROLEUM CONTAINING TAR

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UDC 532.535.2:532.135

A study is made on nonlinear-wave propagation in petroleum containing tar.

Recent studies have shown that high-viscosity petroleum containing much tar shows relaxation behavior [1, 2], which is due to clusters consisting of hundreds or more macromolecules. Such a medium resembles a conglomerate material [3] in having local deformation viscosity due to the compressibility and the elasticity of these particles, which leads to pressure relaxation. In the propagation of a nonstationary wave in such a medium, there may be an effect from the spread in the impact momentum, as has been found in experiments [4].

The following is a system of equations describing the planar one-dimensional motion of such a medium, which includes the equations of continuity and motion together the Tait equation for each phase [5]

$$\frac{\partial\rho_1}{\partial t}+\frac{\partial\rho_1 v_1}{\partial x}=0,\ \frac{\partial\rho_2}{\partial t}+\frac{\partial\rho_2 v_2}{\partial x}=0,$$

$$\rho_1=\rho_1^0\alpha_1,\ \rho_2=\rho_2^0\alpha_2,\ \alpha_1+\alpha_2=1,\ \rho_1\frac{d_1 v_1}{dt}=-\alpha_1\frac{\partial p_1}{\partial x}-F_\mu,$$

$$\rho_2\frac{d_2 v_2}{dt}=-\alpha_2\frac{\partial p_1}{\partial x}+F_\mu,\ F_\mu=\alpha_{01}\alpha_{02}K_\mu(v_2-v_1), \tag{1}$$

$$p_i=\frac{\rho_{0i}^0 c_i^2}{n_i}\left[\left(\frac{\rho_i^0}{\rho_{0i}^0}\right)^{n_i}-1\right],\ \frac{d_i}{dt}=\frac{\partial}{\partial t}+v_i\frac{\partial}{\partial x},\ i=1,\ 2.$$

Ufa Petroleum Institute. Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 51, No. 2, pp. 240-243, August, 1986. Original article submitted June 28, 1985.

0022-0841/86/5102-0927 $12.50